Introduction to Social Network Analysis

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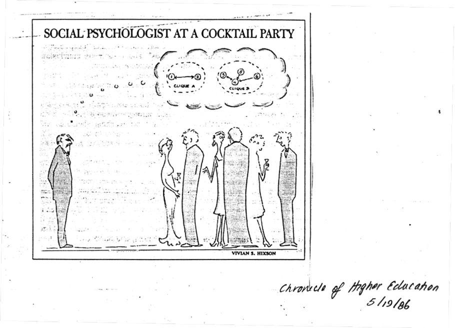

Think Formally

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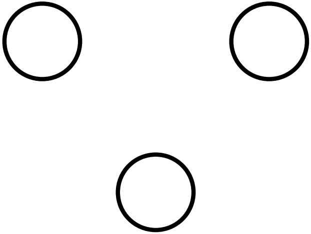

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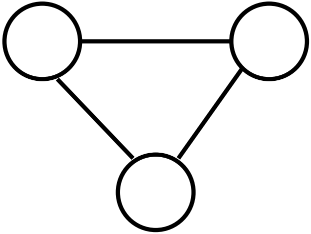

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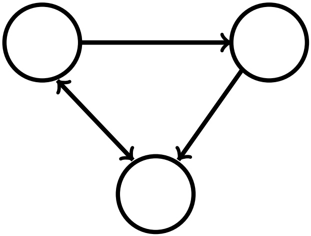
A network is not just a metaphor: it is a precise, mathematical construct of nodes (vertices, actors) N and edges (ties, relations) E that can be directed



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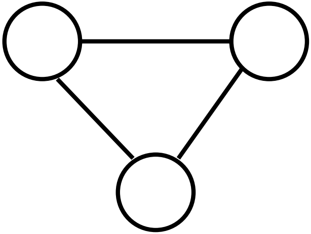

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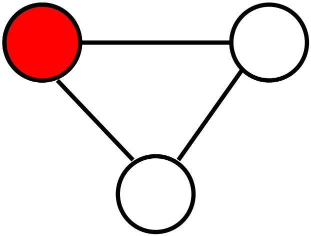

Think Formally

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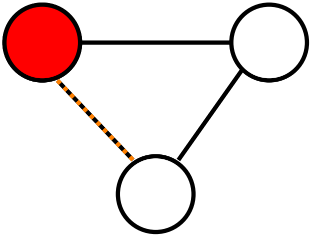
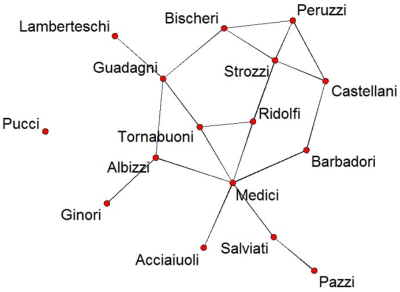


Network Intuition



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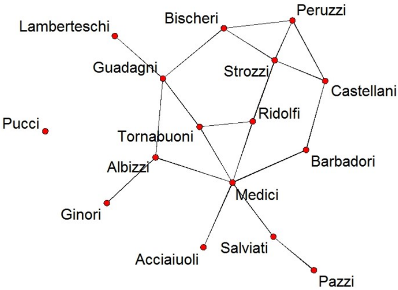

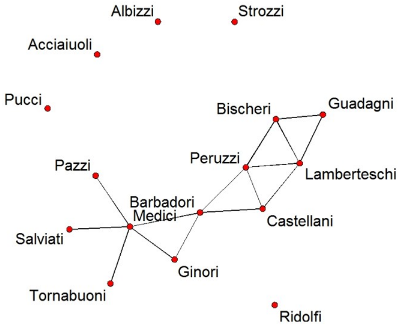

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- Cannot simply use existing statistical methods
- The whole point is that observations are interdependent
- Want to explicitly model these interdependencies

Kinds of Network Relations

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- Interaction (eg communication)

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Types of Actors

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- Individuals

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- Individuals
- Collectives or aggregates
  - Households
  - Organizations
  - Countries

Types of Actors

- Individuals
- Collectives or aggregates
  - Households
  - Organizations
  - Countries
- Other units
  - Objects
  - Locations
  - Beliefs

Types of Network Data

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- ‘Whole’ network data

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  - graphs and digraphs

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- 'Whole' network data
  - graphs and digraphs
  - two-mode or bipartite graphs
- Sampled networks
  - ego networks
  - respondent-driven sampled data (aka link-trace data, depth or breadth first searches)
  - somewhere in-between – start with a sample, and ask respondents for additional 'waves' of data until no new nodes are found

Boundary Specification

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- Methodologically defined based on data collection protocol – be careful!
  - People communicating via a specific bulletin board, radio channel, twitter hashtag

Sampling

All kinds of reasons for sampling

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- when population is too large

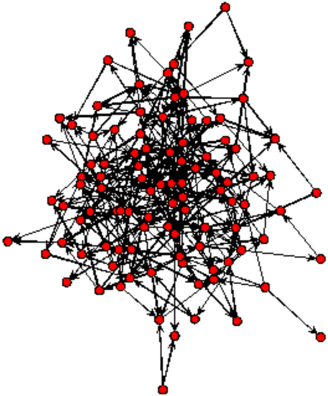
Sampling

All kinds of reasons for sampling

- used when population is hard to find
- when population is too large
- when all the population just doesn't want to respond

Sampling

All kinds of samples



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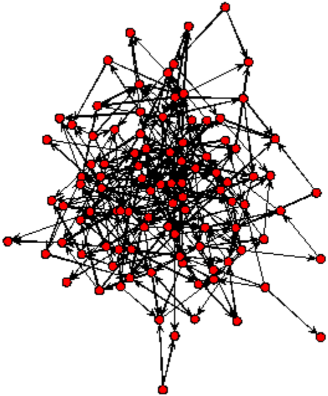

- Ego Networks

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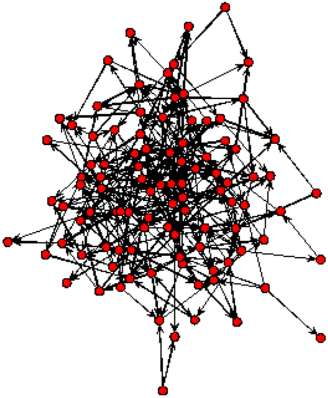
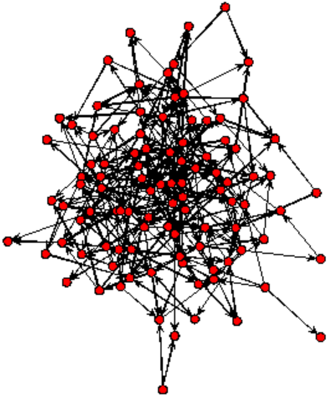


- Ego Networks
- Snowball and Trace sampling

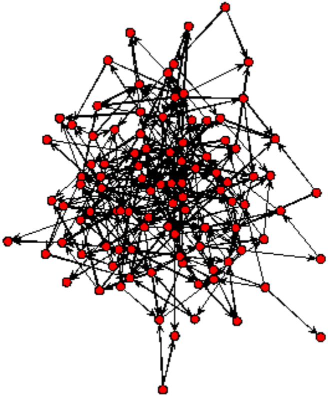
Sampling

All kinds of samples



- Ego Networks
- Snowball and Trace sampling
- Boundary setting

Ego Networks



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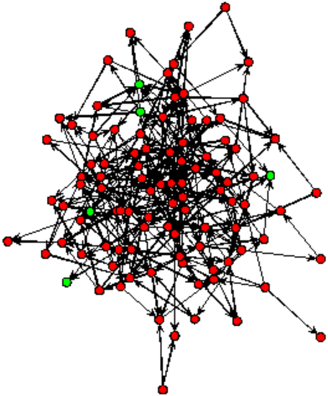
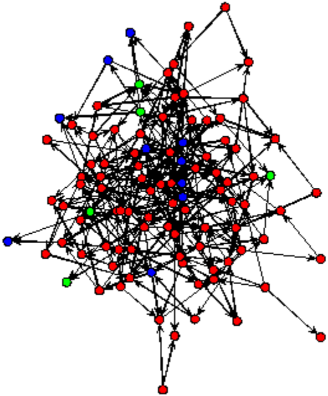


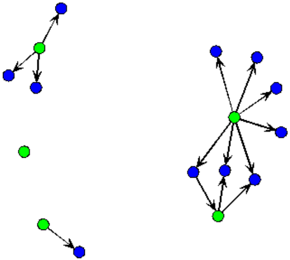
- Randomly select ‘egos’

Ego Networks



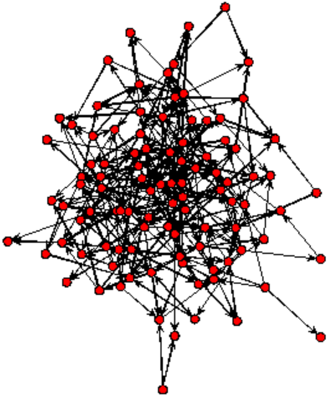
- Randomly select 'egos'
- Find their alters

Ego Networks



- Randomly select ‘egos’
- Find their alters
- Extract the sample made up of the egos and their alters

Trace Networks



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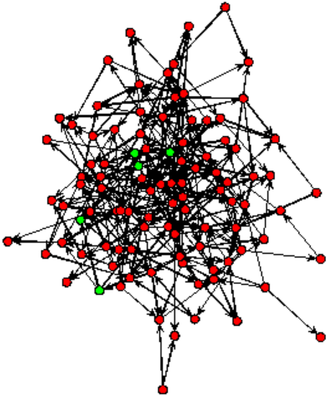

- Randomly select one ‘start’ node

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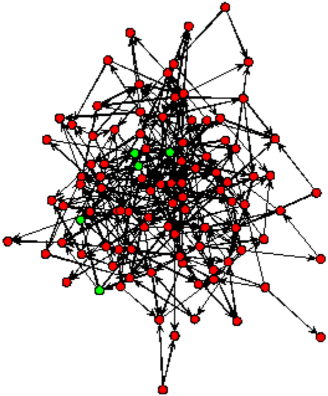

- Randomly select one 'start' node
- Pick one of start's alters

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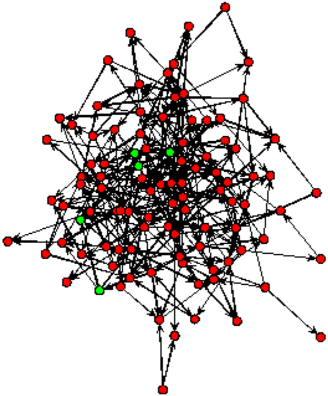

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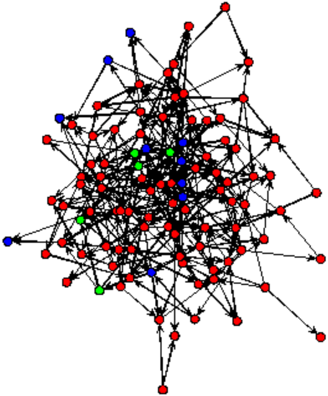

- Randomly select one 'start' node
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- Select all of their alters for the sample

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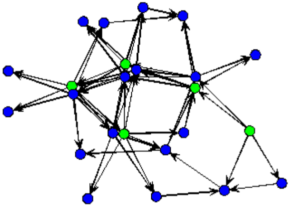

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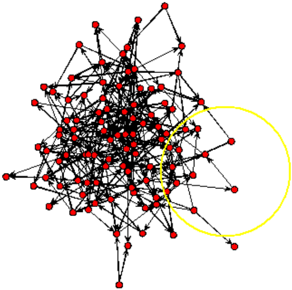

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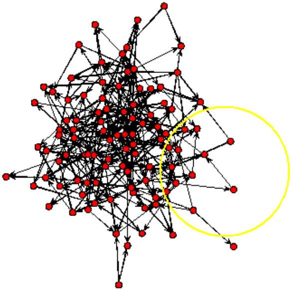

- Set a boundary around some subset of nodes

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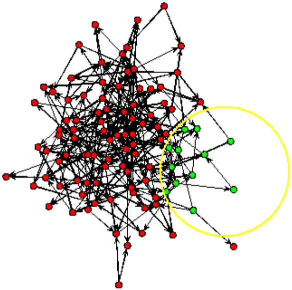

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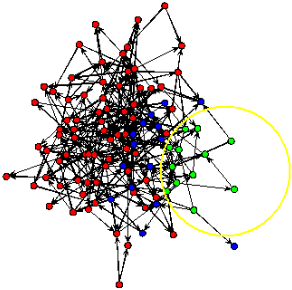

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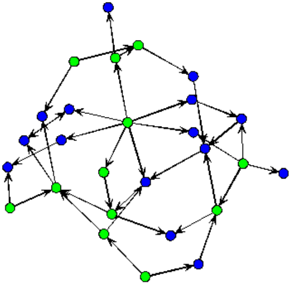

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- Select nodes within the boundary
- Collect their alters
- They are your sample

Data Collection Questions

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  - Roster-based
  - Who do you discuss ‘important questions’ with?
- Missing data

Exercise

- Take a sheet of paper and divide into 3 columns
- Write the names of the last 10 people you spoke to in the first column
- Write their relationship to you in the next column
- Write who introduced you to them in the third

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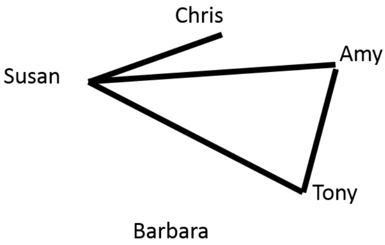
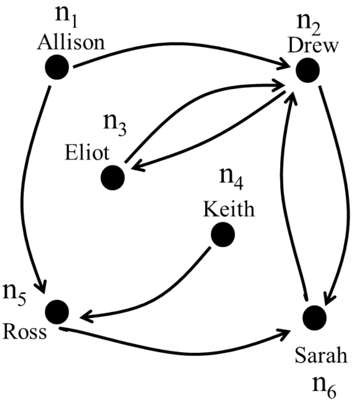
Name	Relationship	Introduced by
Susan	Boss	Travis
Chris	Friend	Jamie
Barbara	Mum	No one

Exercise

- Flip the paper over
- Write each name from the first column spread out on the page
- Draw a line between two people if they know each other



Data Structures



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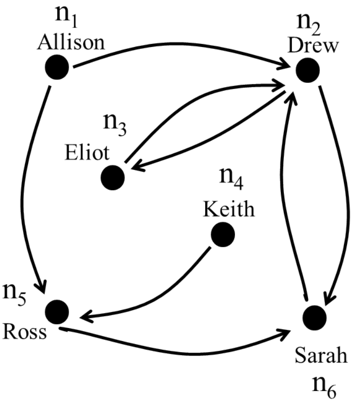

	n_1	n_2	n_3	n_4	n_5	n_6
n_1						
n_2						
n_3						
n_4						
n_5						
n_6						

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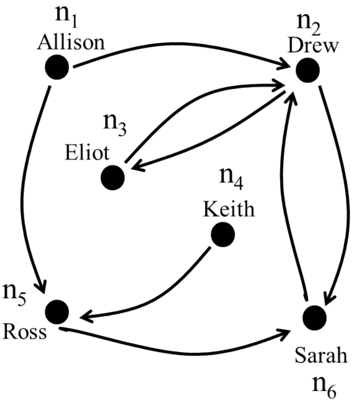

	n_1	n_2	n_3	n_4	n_5	n_6
n_1		1				
n_2						
n_3						
n_4						
n_5						
n_6						

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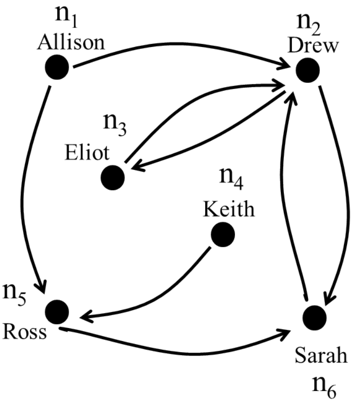

	n_1	n_2	n_3	n_4	n_5	n_6
n_1		1			1	
n_2						
n_3						
n_4						
n_5						
n_6						

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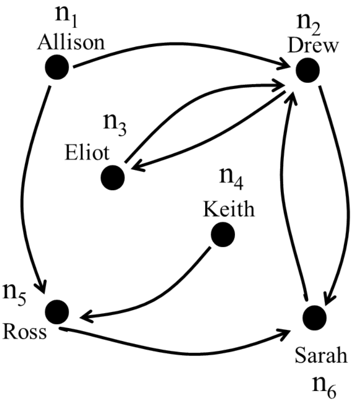

	n_1	n_2	n_3	n_4	n_5	n_6
n_1		1			1	
n_2			1	1		
n_3		1				
n_4					1	
n_5						1
n_6		1				

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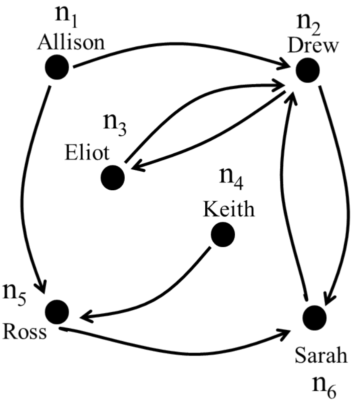

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	-	1			1	
n_2		-	1	1		
n_3		1	-			
n_4				-	1	
n_5					-	1
n_6		1				-

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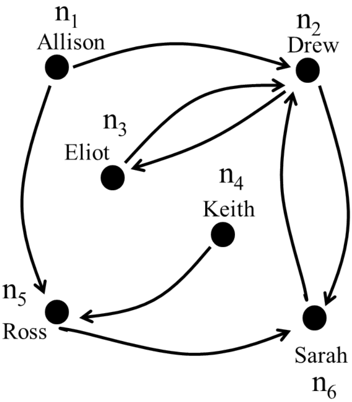

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	-	1	0	0	1	0
n_2	0	-	1	1	0	0
n_3	0	1	-	0	0	0
n_4	0	0	0	-	1	0
n_5	0	0	0	0	-	1
n_6	0	1	0	0	0	-

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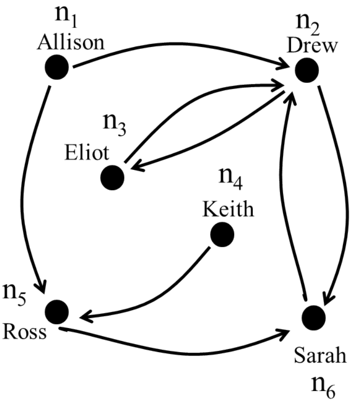

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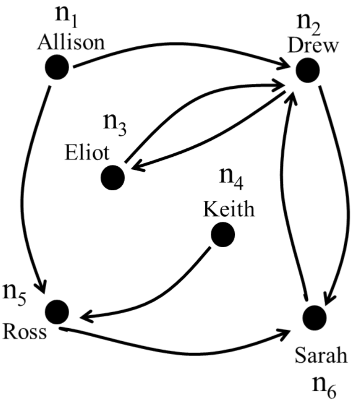
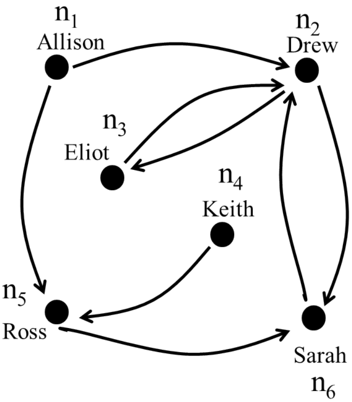


Sender	Receiver	Weight

Data Structures



Sender	Receiver	Weight
n_1	n_2	1

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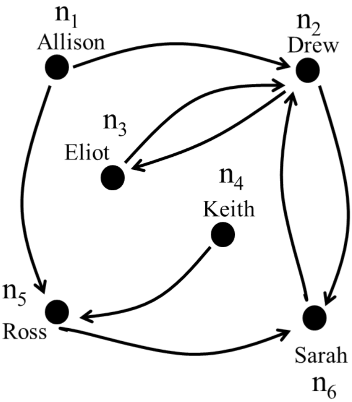

Sender	Receiver	Weight
n_1	n_2	1
n_1	n_5	1
n_2	n_3	1
n_2	n_6	1
n_3	n_2	1
n_4	n_5	1
n_5	n_6	1
n_6	n_2	1

Network Objects

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  - Similarly for edge attributes (`%e%`)
  - And network level attributes (`%n%`)

Network Objects

```
Network attributes:
 vertices = 18
 directed = TRUE
 hyper = FALSE
 loops = FALSE
 multiple = FALSE
 bipartite = FALSE
 total edges= 54
   missing edges= 0
   non-missing edges= 54

Vertex attribute names:
   Group vertex.names

Edge attribute names:
   Order
> |
```

Example

- Network from the Climate Constituencies Project

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- Looks at Obama's Clean Power Plan in 4 States and National levels

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https://www.youtube.com/watch?v=KHIWUgeNB7E&list=PLA5s0tCtlX930LrwGp05mc9ao2jt2v1Dc&index=3

Examples

Informal Networks: The Company Behind the Chart by David Krackhardt and Jeffrey R. Hanson (Harvard Business Review July-August 1993)

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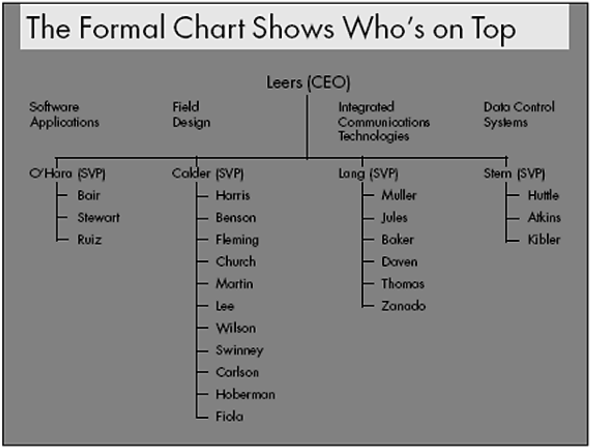

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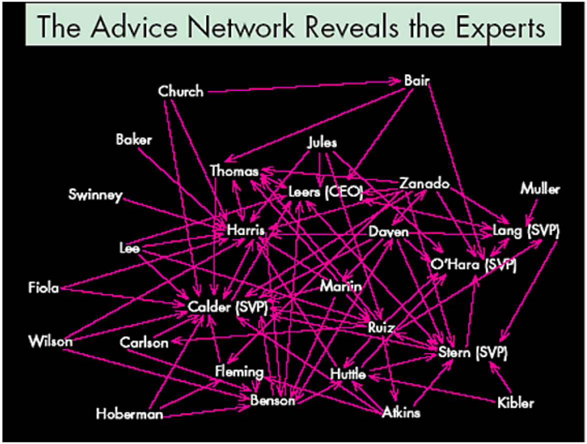

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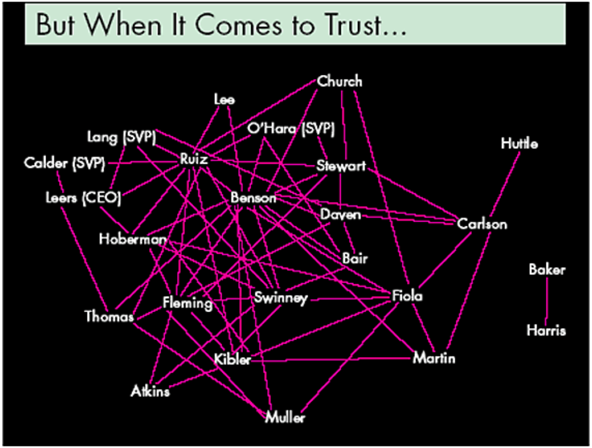

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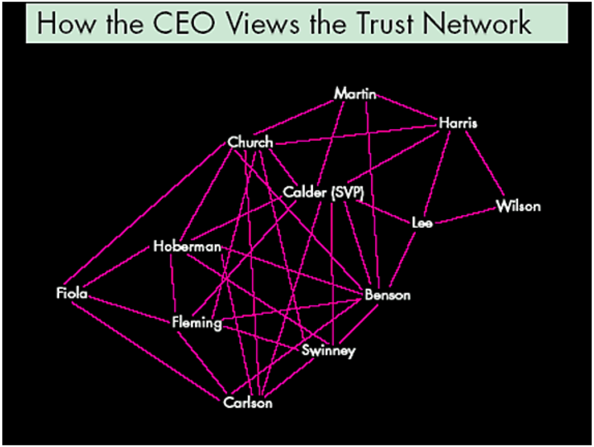

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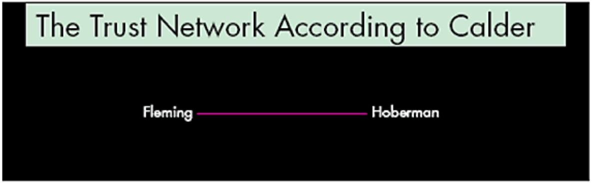
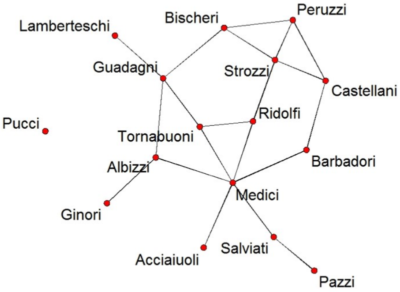


Descriptives



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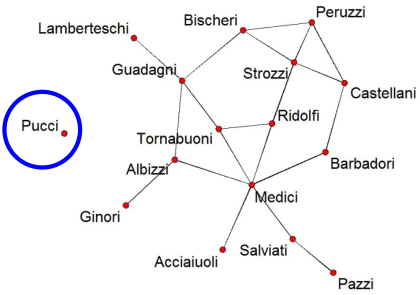
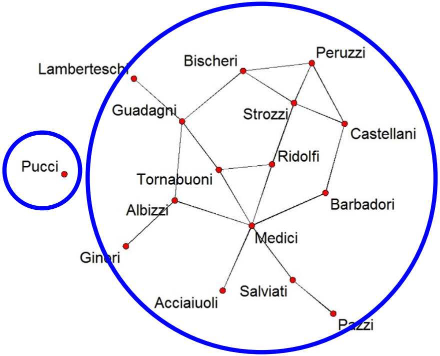


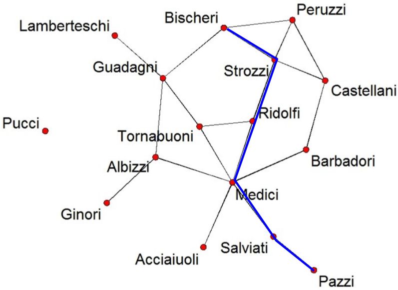
- One isolate

Descriptives



- One isolate
- Two components

Descriptives



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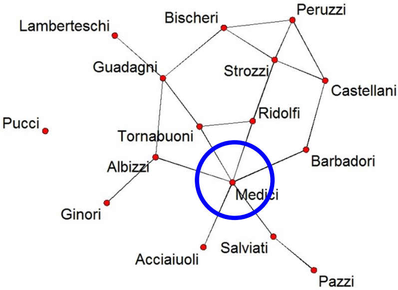
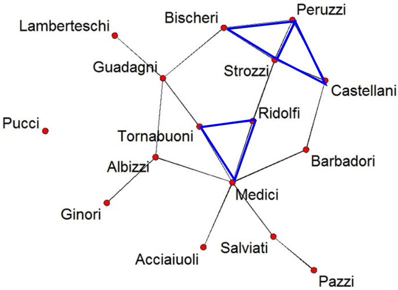
- Two components
- Diameter is 5

Descriptives



- One isolate
- Two components
- Diameter is 5
- Medici is most popular

Descriptives



- One isolate
- Two components
- Diameter is 5
- Medici is most popular
- Three triads

Degree

For each node, its degree is

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- the number of nodes adjacent to it

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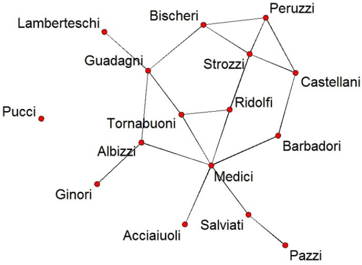

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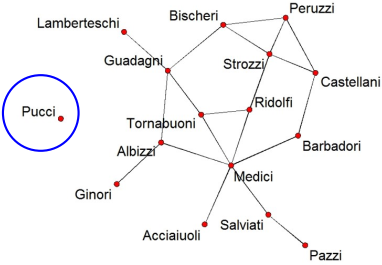

- Pucci has degree 0

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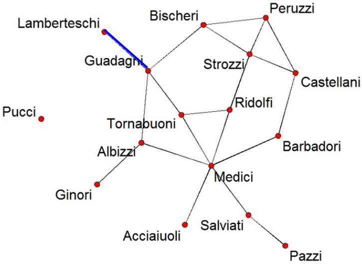

- Pucci has degree 0
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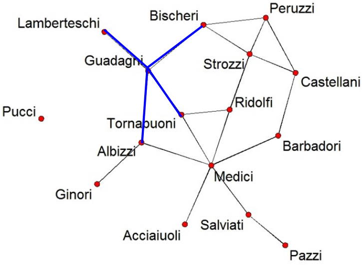

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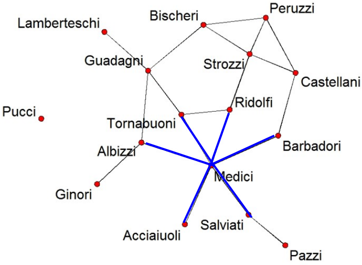
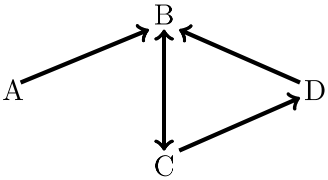


- Pucci has degree 0
- Lamberteschi has degree 1
- Guadagni has degree 4
- Medici has degree 6

Directed Degree

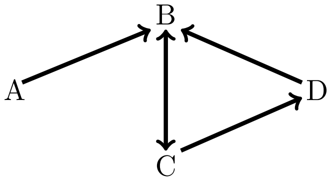
In directed graphs,



Directed Degree

In directed graphs,

- *Indegree* indicates the number of received ties



Directed Degree

In directed graphs,

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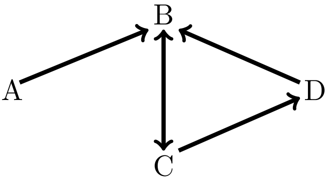

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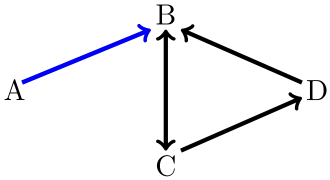

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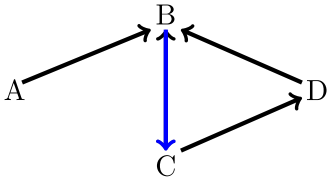

- A has 1 outdegree
- B has 1 outdegree

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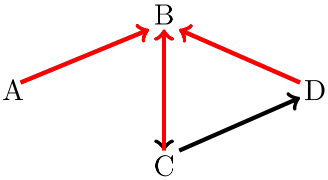

- A has 1 outdegree
- B has 1 outdegree and 3 indegree

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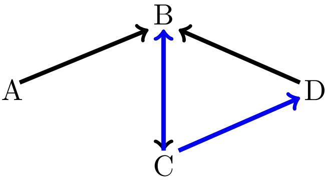

- A has 1 outdegree
- B has 1 outdegree and 3 indegree
- C has 2 outdegree

Directed Degree

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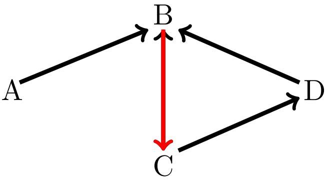

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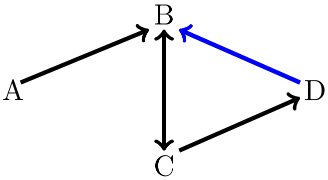

- A has 1 outdegree
- B has 1 outdegree and 3 indegree
- C has 2 outdegree and 1 indegree
- D has 1 outdegree

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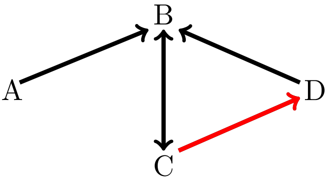

- A has 1 outdegree
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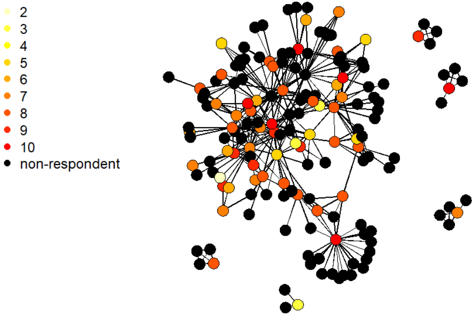
- C has 2 outdegree and 1 indegree
- D has 1 outdegreeand 1 indegree

Connecting Communities



Betweenness Centrality

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Proportion of shortest paths between all other pairs of nodes that the given node lies on

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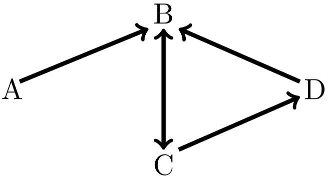

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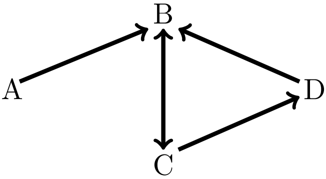

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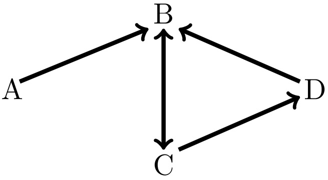

- A sits on no paths between others
- B sits on some paths:

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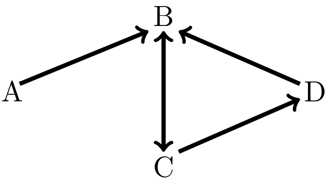

- A sits on no paths between others
- B sits on some paths:
  - A→ C
  - A→ D
  - C→ D
  - D→ C
  - C→ A
  - D→ A

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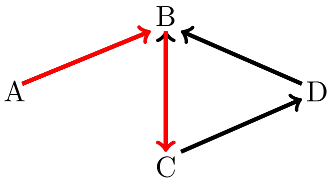

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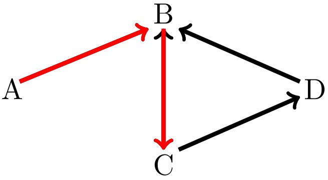

- A sits on no paths between others
- B sits on some paths:
  - A→ C = 1
  - A→ D
  - C→ D
  - D→ C
  - C→ A
  - D→ A

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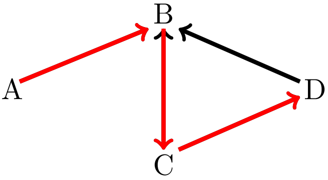

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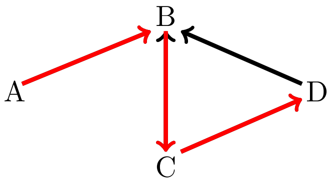

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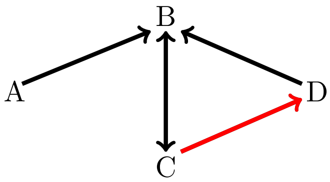

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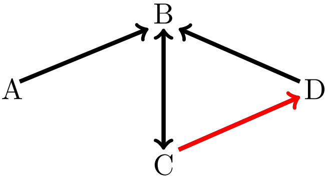

- A sits on no paths between others
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  - A→ C = 1
  - A→ D = 1
  - C→ D = 0
  - D→ C
  - C→ A
  - D→ A

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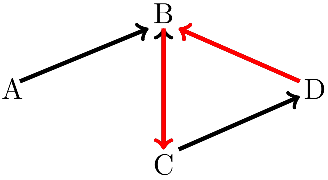

- A sits on no paths between others
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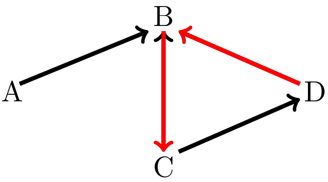

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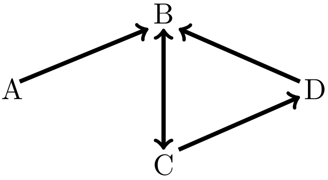

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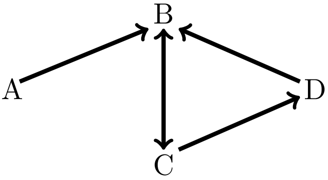

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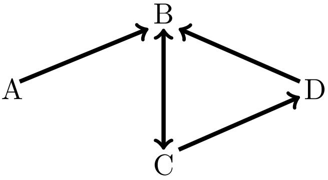

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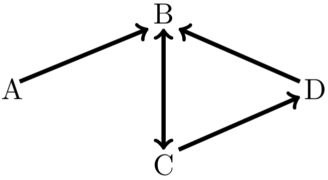
  - D$\rightarrow$ C = 1
  - C$\rightarrow$ A = 0
  - D$\rightarrow$ A

Betweenness Centrality

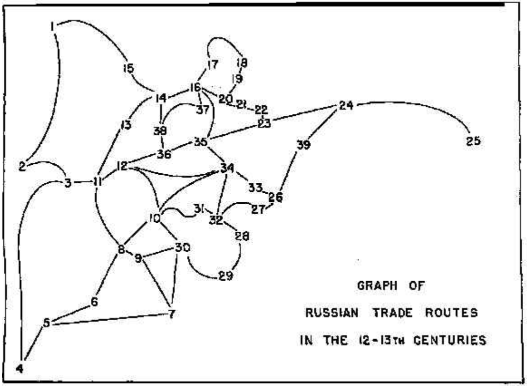
Proportion of shortest paths between all other pairs of nodes that the given node lies on



- A sits on no paths between others
- B sits on some paths:
  - A→ C = 1
  - A→ D = 1
  - C→ D = 0
  - D→ C = 1
  - C→ A = 0
  - D→ A = 0

Betweenness Centrality

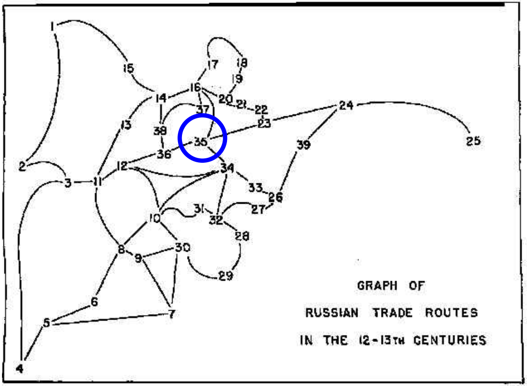
Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



Forrest Pitts 1978 "The River Trade Network of Russia, Revisited"

Betweenness Centrality

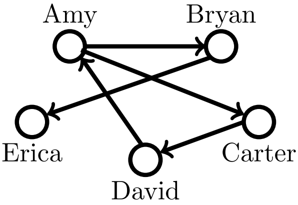
Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



Forrest Pitts 1978 “The River Trade Network of Russia, Revisited”

Page Rank and Eigenvector Centrality

Page Rank



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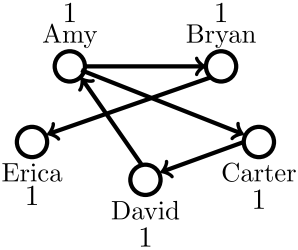

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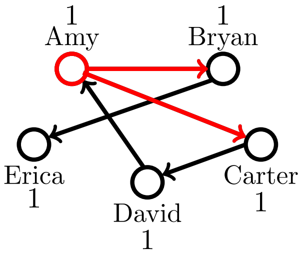

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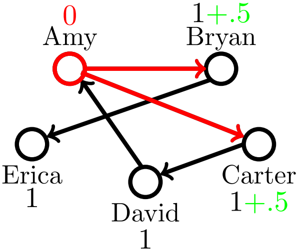

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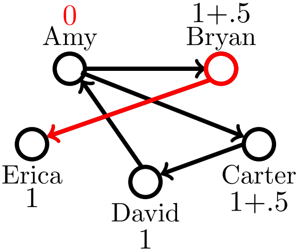

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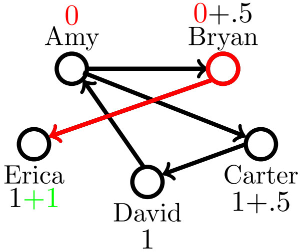

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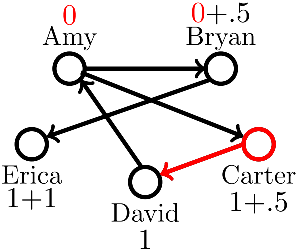

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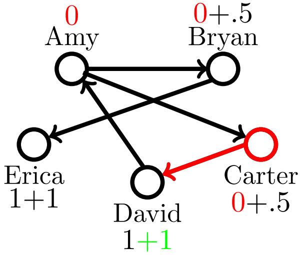

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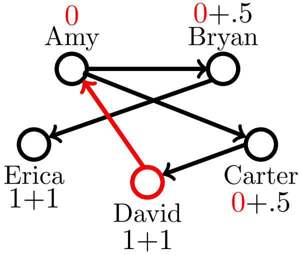

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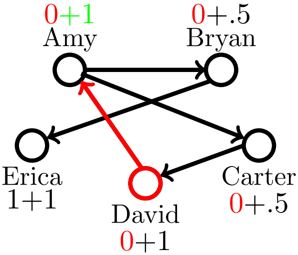

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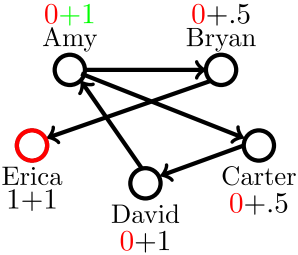

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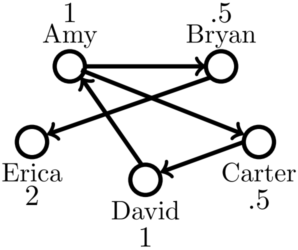

Page Rank Algorithm

```
Let v = [1, 1, 1, 1, ... 1].
Repeat 100 times {
 Let w = [0, 0, 0, 0, ... 0].
 For each person i in the social network
  For each friend j of i
   Set w[j] = w[j] + v[i].
 Set v = w.
}
Let S be the sum of the entries of v.
Divide each entry of v by S.
```

Centralization

Centralization

- Extent to which centrality is concentrated on a single vertex

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- Calculated as the sum of the differences between each node's centrality score and the maximum score

Centralization

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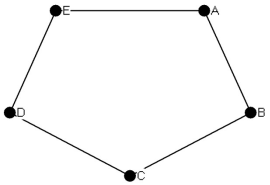
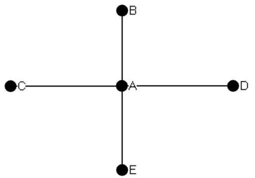
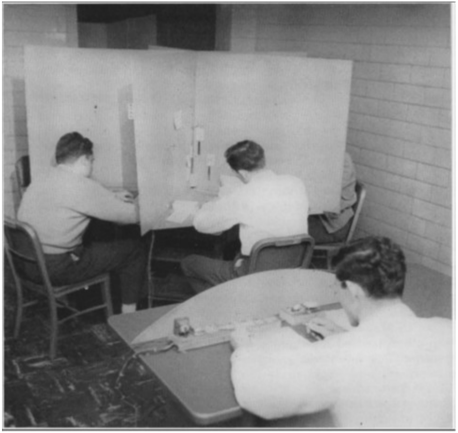
- Calculated as the sum of the differences between each node's centrality score and the maximum score
- Most centralized structure is usually a star network



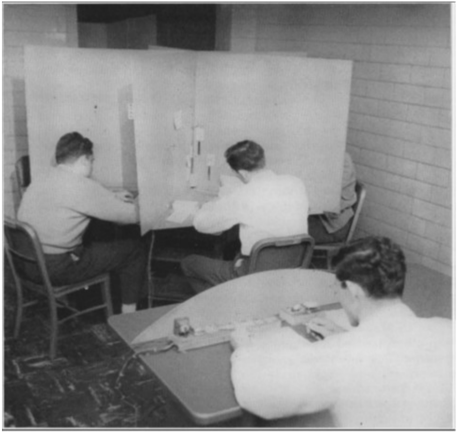


Bavelas Experiments



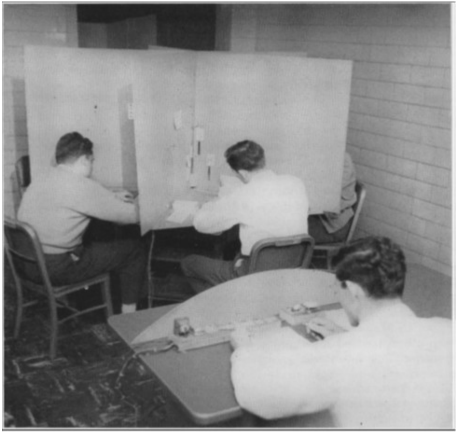
Bavelas Experiments

- People in positions passed messages to one another to solve a problem

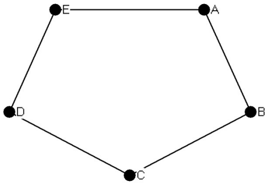


Bavelas Experiments

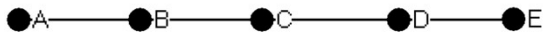
- People in positions passed messages to one another to solve a problem
- Studied the effect of structure on
  - Efficiency
  - Leadership
  - Satisfaction



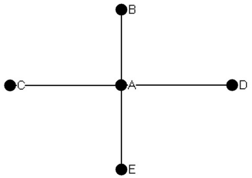
Bavelas Experiments



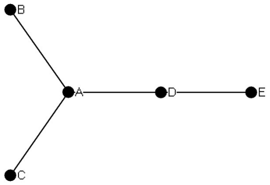
Circle



Line

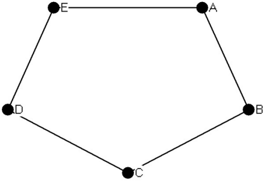


Star

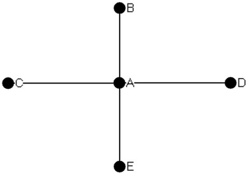


Y

Bavelas Experiments

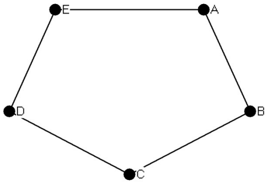


Circle



Star

Bavelas Experiments



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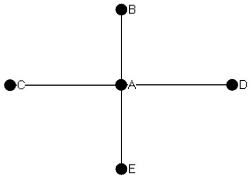
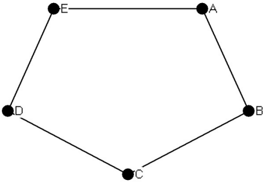


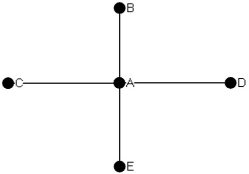
Star

- Slowest to completion

Bavelas Experiments



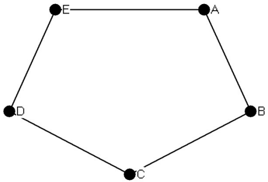
Circle



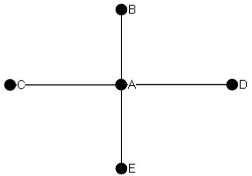
Star

- Slowest to completion
- Most errors

Bavelas Experiments



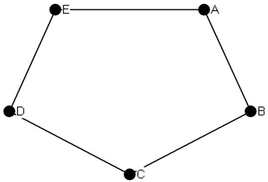
Circle



Star

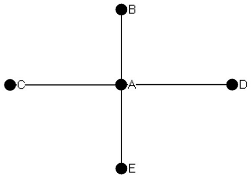
- Slowest to completion
- Most errors
- Most satisfied

Bavelas Experiments



Circle

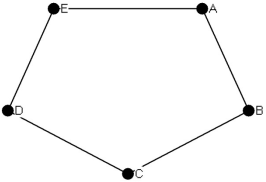
- Slowest to completion
- Most errors
- Most satisfied



Star

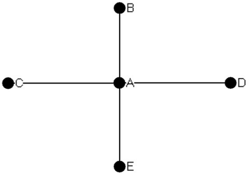
- Fastest

Bavelas Experiments



Circle

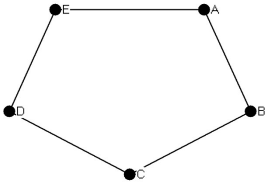
- Slowest to completion
- Most errors
- Most satisfied



Star

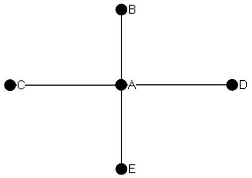
- Fastest
- Fewest errors

Bavelas Experiments



Circle

- Slowest to completion
- Most errors
- Most satisfied



Star

- Fastest
- Fewest errors
- Most dissatisfied

Dyad Census

Dyad Census

Mutual (M)

Dyad Census

Mutual (M)

Assymetric (A)

Dyad Census

Mutual (M)

Assymetric (A)

Null (N)

Reciprocity

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- Dyadic: the proportion of dyads that are symmetric
  $$\frac{M+N}{M+A+N}$$

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- Dyadic: the proportion of dyads that are symmetric $\frac{M+N}{M+A+N}$
- Dyadic non-null: the proportion of non-null dyads that are reciprocal $\frac{M}{M+A}$

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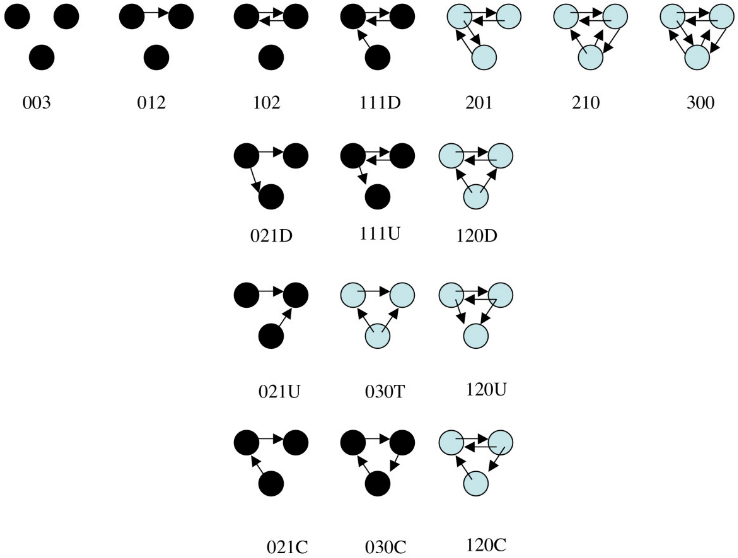
- Dyadic: the proportion of dyads that are symmetric $\frac{M+N}{M+A+N}$
- Dyadic non-null: the proportion of non-null dyads that are reciprocal $\frac{M}{M+A}$
- Edgewise: $\frac{2*M}{2*M+A}$

Triad Census

Triad Census

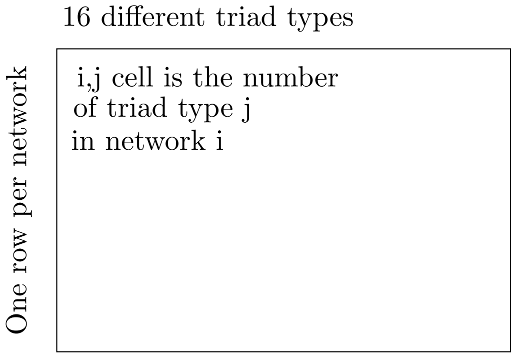


Triad Census

16 different triad types

One row per network

Triad Census



Multidimensional Scaling

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- related to Principal Components Analysis, Factor Analysis, etc.

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- Reduces multidimensional data on similarities or differences to lower dimensions – data simplification

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- Reduces multidimensional data on similarities or differences to lower dimensions – data simplification
- Good visualizations – more similar networks should be closer together

Triad Census

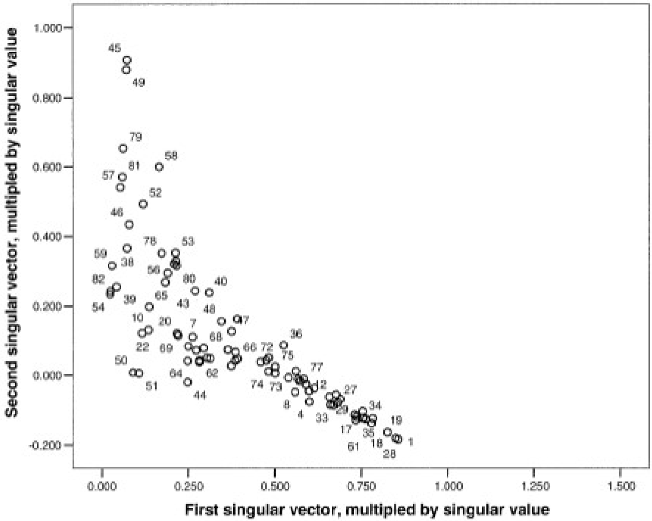
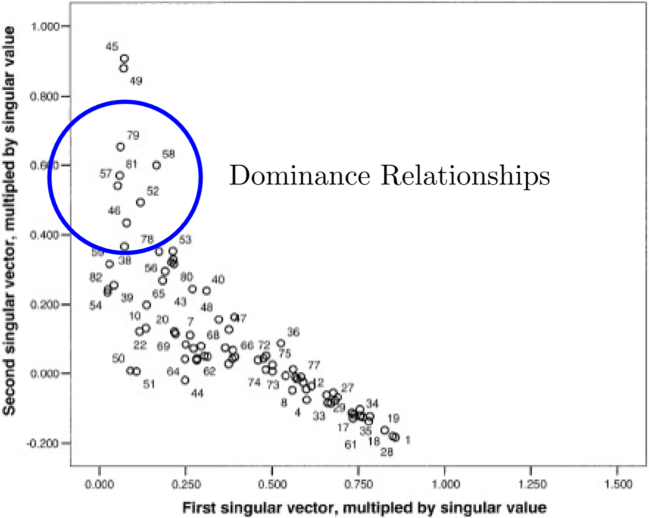


FIGURE 2. Singular value decomposition of triad census array, first two left singular vectors, multiplied by singular values, $N = 82$ networks.

Triad Census



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Triad Census

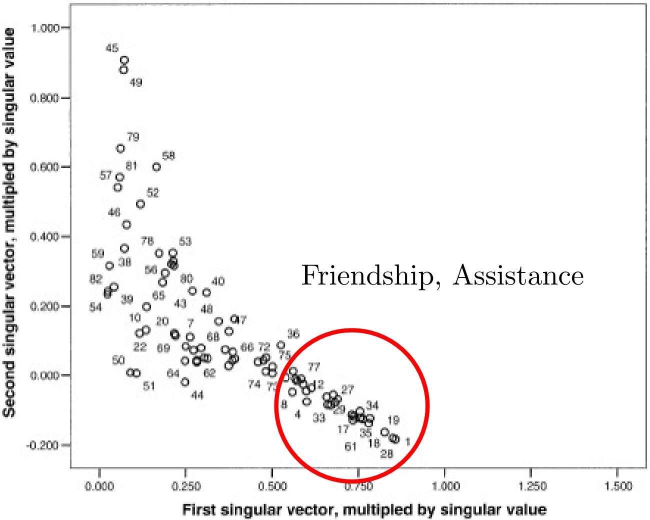
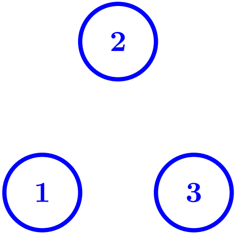


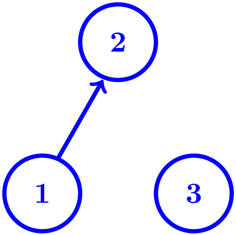
FIGURE 2. Singular value decomposition of triad census array, first two left singular vectors, multiplied by singular values, $N = 82$ networks.

Transitivity

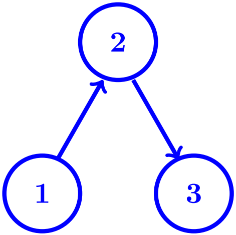
Transitivity



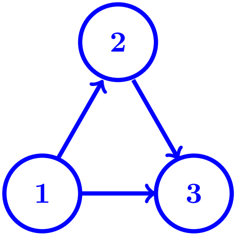
Transitivity



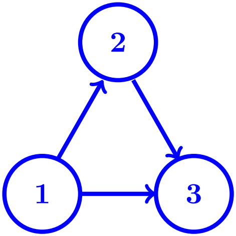
Transitivity



Transitivity

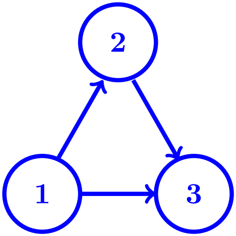


Transitivity



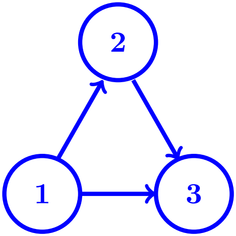
- Usually calculated as the fraction of completed two-paths

Transitivity



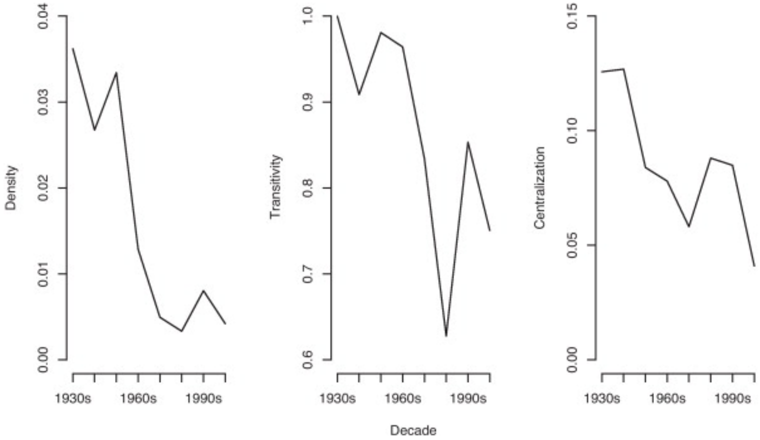
- Usually calculated as the fraction of completed two-paths
- Related to Grannovetter's 'forbidden triad'

Transitivity



- Usually calculated as the fraction of completed two-paths
- Related to Grannovetter's 'forbidden triad'
- Can be directed or undirected

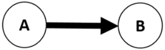
Example



Janet Box-Steffensmeier and Dino Christenson
"The evolution and formation of amicus curiae networks"
Social Networks 2012

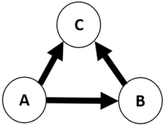
Other Triads

a)



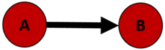
A sends information to B.

c)



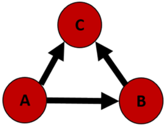
Transitive triad such that A sends information to B and C, and B also sends information to C. The smallest example of a 'chamber.'

b)



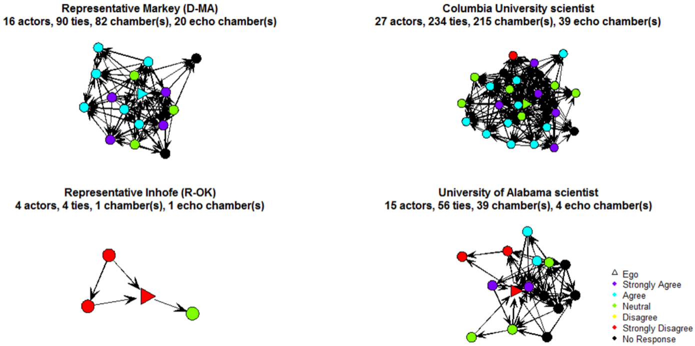
A and B agree so A's information echoes B's understanding.

d)



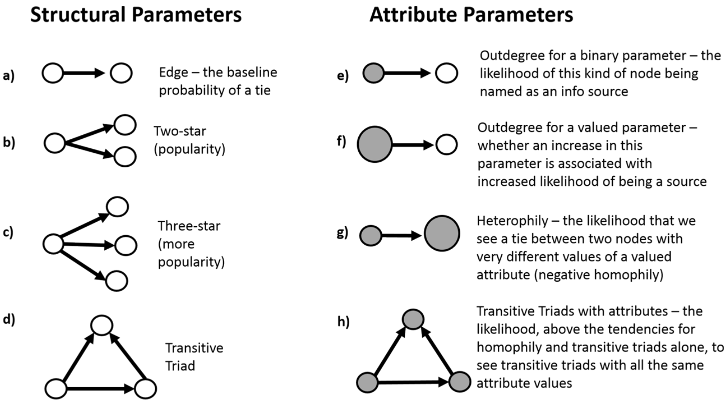
A transitive triad where each actor already holds the same position – an echo chamber.

Other Triads

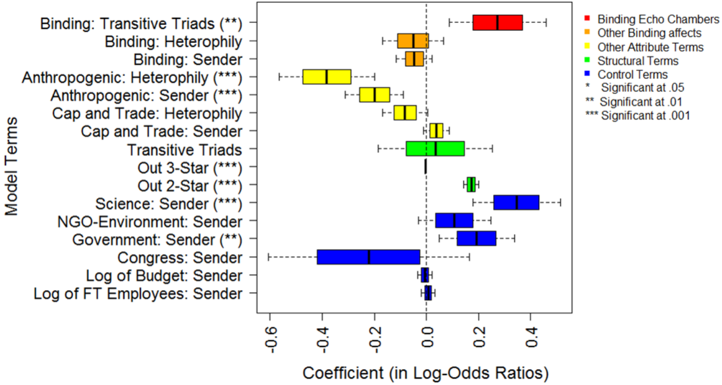


Other Triads

Other Triads



Other Triads



Forbidden Triad or Structural Hole?

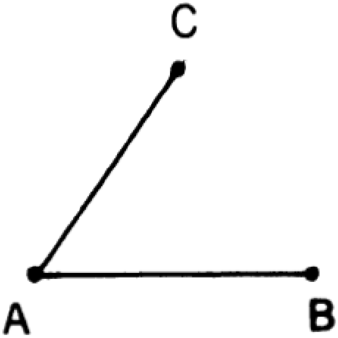


FIG. 1.—Forbidden triad

Forbidden Triad or Structural Hole?

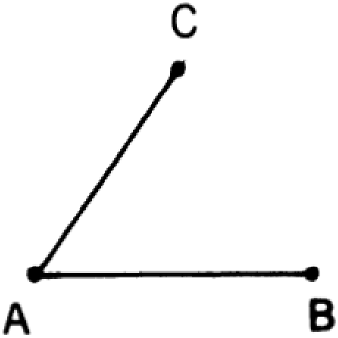
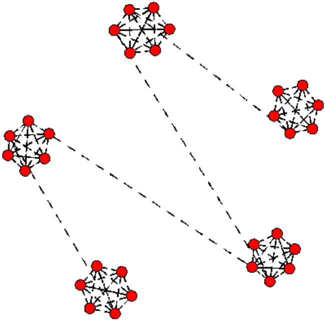


FIG. 1.—Forbidden triad

- Granovetter, Mark S. 1973. “The Strength of Weak Ties”

Forbidden Triad or Structural Hole?



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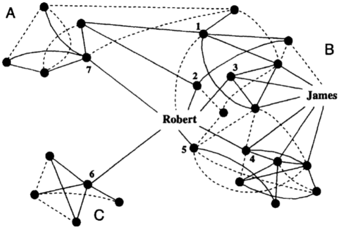
Forbidden Triad or Structural Hole?



- Burt, Ronald S. 2004. "Structural Holes: The Social Structure of Competition"

Extensions

Attributes!

Extensions

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- Extension based on node attributes: Brokerage
- Extension based on edge attributes: Structural Balance

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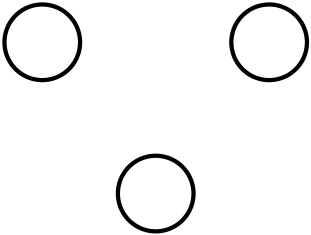
- Brokers play a crucial role in knitting together diverse groups of people, organizations, parties
- Brokers can gain a lot – early access to information, prestige
- But can also be distrusted by everyone

Brokerage: Formal Concept

In a network N with edges E,



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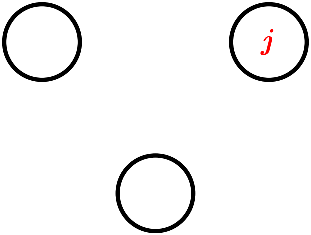

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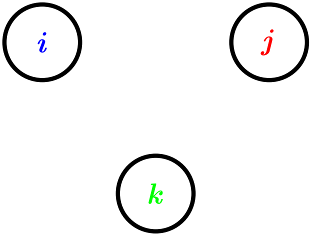

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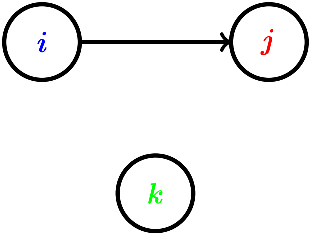

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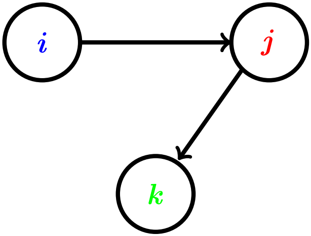

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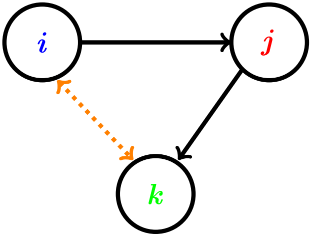

Brokerage: Formal Concept

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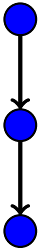
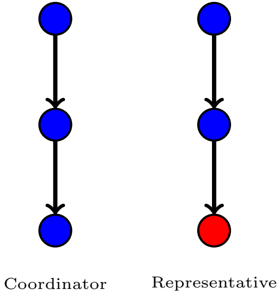
- added a vertex attribute component
- compared empirical brokerage counts to counts from random graphs conditioned on the number of edges

Brokerage: One Mode



Coordinator

Brokerage: One Mode



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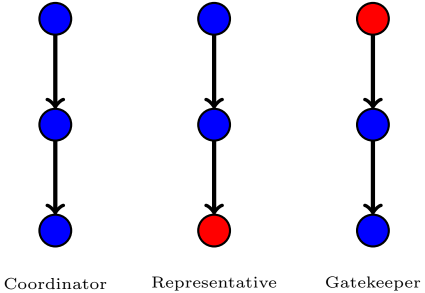

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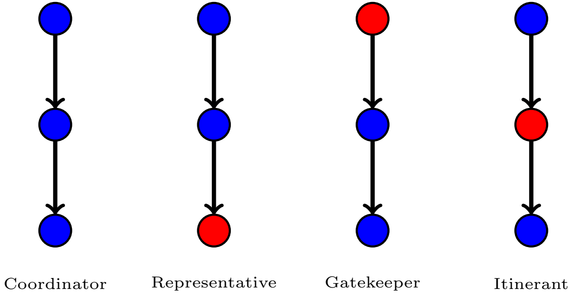

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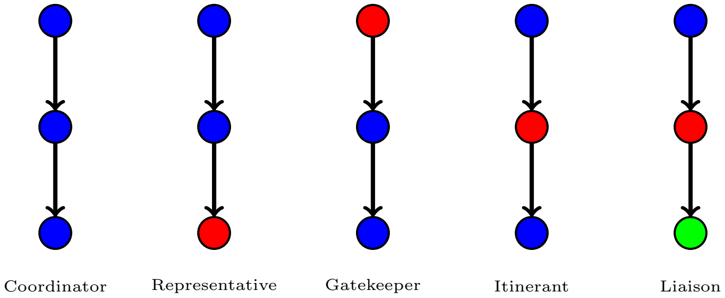

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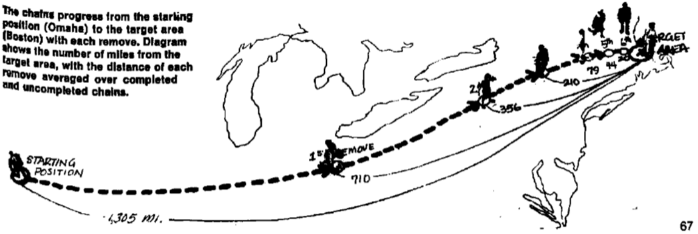
- non-governmental organizations were found to have more influence when they held any type of brokerage position
- governmental organizations gained influence only when they held "outsider" brokerage roles in itinerant and liaison chains

Small Worlds

What are the characteristics of real world?

Small Worlds

Stanley Milgram "The Small World Problem", *Psychology Today*, vol. 1, no. 1, May 1967, pp61-67



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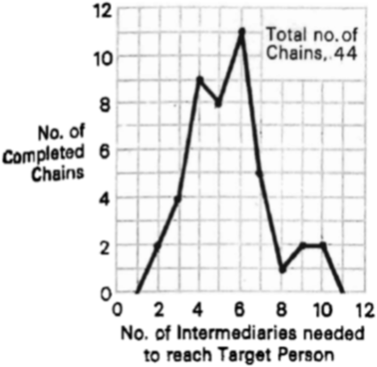

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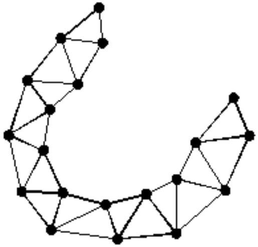

Watts-Strogatz Model

- high clustering coefficient
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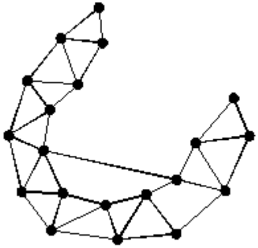
- starts with a lattice structure
- randomly re-wires ties



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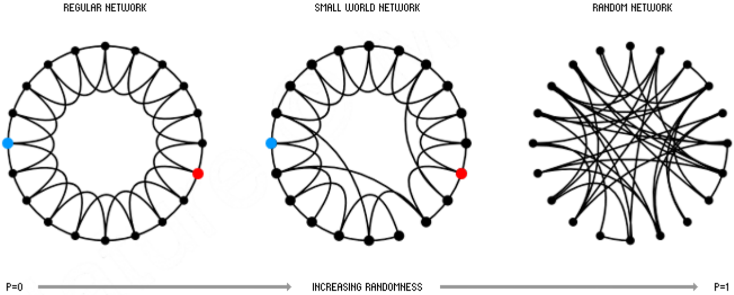

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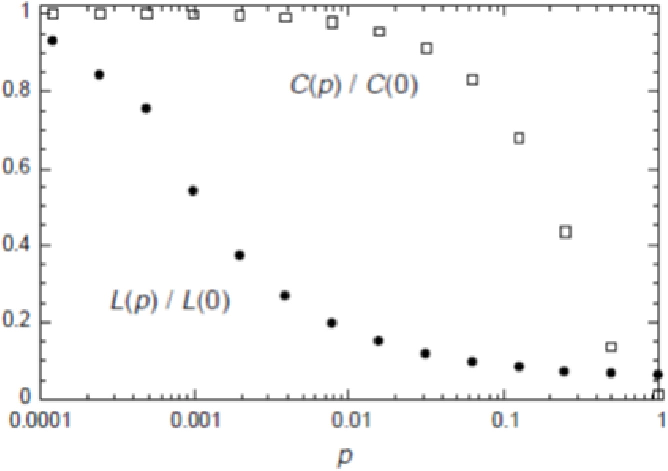

Barabasi-Albert Model

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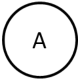

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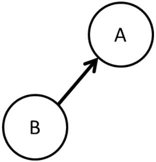

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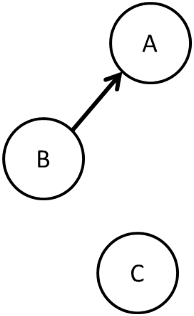

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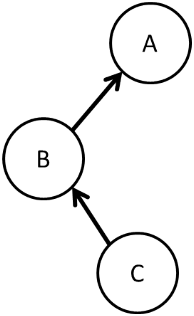

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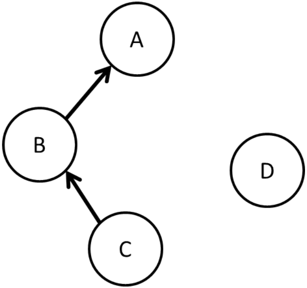

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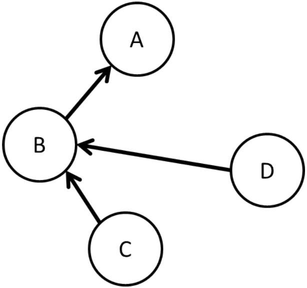

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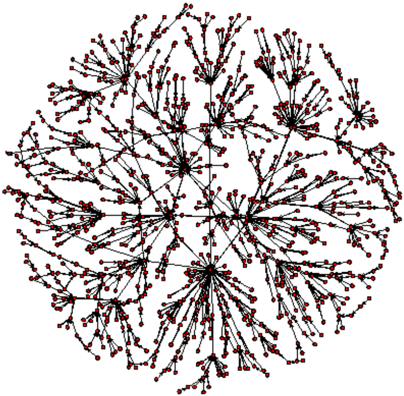

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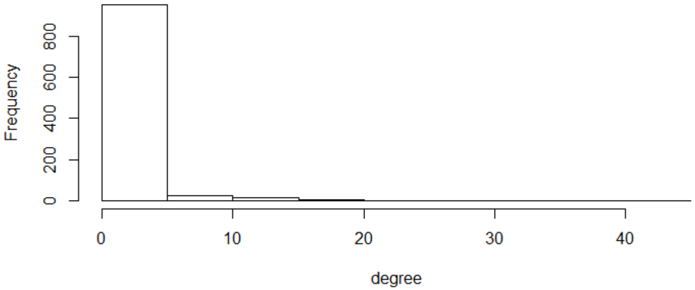

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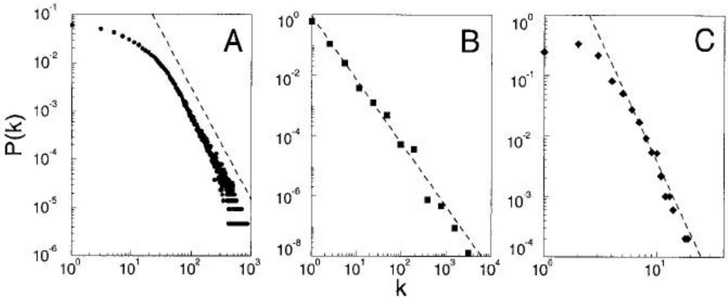

Fig. 1. The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. **(B)** WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (6). **(C)** Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{actor} = 2.3$, (B) $\gamma_{www} = 2.1$ and (C) $\gamma_{power} = 4$.

Barabási, A.-L.; R. Albert (1999). "Emergence of scaling in random networks". *Science*